Response of a Cylindrical Shell to the Sudden Breakdown of a Ring Stiffener

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Introduction

IN recent years there has been a growing trend in aeronautical engineering to design structures using the concept of damage tolerance in order to achieve high reliability and long service life of the structures. The philosophy of damage tolerance recognizes that in every structure there exist inherent defects which may grow to sizes mortal to the structure during service, and demands that the structure must still be able to sustain relatively heavy loads in case one of its components has collapsed. Therefore, to study the dynamic response of a structure to the sudden breakdown of one of its components is of engineering significance and deserves attention.

In this Note, the corresponding problem of an infinitely long circular cylindrical shell with a single ring stiffener submitted to the action of a uniform pressure (a simple idealization of an airplane pressure cabin or a reinforced boiler) will be considered. The sudden breakdown of the ring stiffener takes place with the simultaneous vanishing of the radial line force, uniformly distributed along the attachment section and applied to the shell by the ring stiffener (see Fig. 1). So, by a simple superposition argument, it can be seen that the essential part of the problem under consideration is equivalent to the dynamic response problem of an infinitely long circular cylindrical shell subjected to the radial line force $H(t)\delta(t)$, distributed symmetrically with respect to its axis and acting along its center section (see Fig. 1). $H(t)$ and $\delta(x)$ are the Heaviside unit function and Dirac delta function, respectively. This reduced problem is a dynamic counterpart to the static one that has been discussed in detail in Ref. 1 and is treated in the next sections.

The Problem and the Formal Solutions

Consider an infinitely long circular cylindrical shell of radius a, its longitudinal axis coinciding with the x axis. Suppose a radial line force $H(t)\delta(0)$, uniformly distributed along the cross section of $x=0$, is applied to the shell and it is desired to find its internal moment and displacement for $t>0$, $-\infty<x<\infty$. On account of the symmetry with respect to the shell axis of the problem, the governing equation for the radial displacement w of the shell can be written as[1]

$$\frac{\partial^4 w}{\partial x^4}+4\beta^4 w+\gamma\frac{\partial^2 w}{\partial t^2}=\frac{H(t)\delta(0)}{D} \qquad (1)$$

where

$$D=\frac{Eh^3}{12(1-\nu^2)},\quad \beta^4=\frac{3(1-\nu^2)}{(ah)^2},\quad \gamma=\frac{12\rho(1-\nu^2)}{Eh^2}$$

h is the shell thickness, and E, ν, and ρ are Young's modulus, Poisson's ratio, and the density of the shell material, respectively.

The solution of Eq. (1), in association with the boundary condition $w(\pm\infty,t)=0$ and the initial condition $w(x,0)=0$, can be obtained by integral transform methods in a straightforward manner. The result is

$$w(x,t)=\frac{1}{D\pi}\int_0^\infty\left\{\frac{1-\cos\{[(4\beta^4+s^4)/\gamma]^{1/2}t\}}{4\beta^4+s^4}\right\}\cos sx\,ds \qquad (2)$$

The bending moment M_x is the shell can be expressed in terms of w in the form[1]

$$M_x(x,t)=-D\frac{\partial^2 w}{\partial x^2}$$

$$=\frac{1}{\pi}\int_0^\infty\left\{\frac{1-\cos\{[(4\beta^4+s^4)/\gamma]^{1/2}t\}}{4\beta^4+s^4}\right\}s^2\cos sx\,ds \qquad (3)$$

However, Eqs. (2) and (3) are formal solutions and further manipulations are needed to obtain explicit forms. This is to be done in the following sections.

Solutions for $w(0,t)$ and $M_x(0,t)$

During the dynamic process, the highest values of w and M_x occur at $x=0$, the cross section to which the load $H(t)\delta(0)$ applies. In this section, exact and explicit formulas for them are derived.

Stationary Solutions

On account of the infinite length of the shell, a stationary state reigns for $t=\infty$ around $x=0$. In this case, the values of

$$\int_0^\infty\left\{\frac{\cos\{[(4\beta^4+s^4)/\gamma]^{1/2}t\}}{4\beta^4+s^4}\right\}s^n ds,\quad n=0,2$$

are obviously zero and the stationary solutions for w and M_x can be easily obtained as

$$w(0,\infty)=\frac{1}{D\pi}\int_0^\infty\frac{ds}{4\beta^4+s^4}=\frac{1}{8\beta^3 D} \qquad (4)$$

$$M_x(0,\infty)=\frac{1}{\pi}\int_0^\infty\frac{s^2 ds}{4\beta^4+s^4}=\frac{1}{4\beta} \qquad (5)$$

The above formulas agree with those given in Ref. 1 for the static case.

Transient Solutions

Now consider the transient case. Letting

$$J(t)=\frac{1}{D\pi}\int_0^\infty\frac{\cos\{[(4\beta^4+s^4)/\gamma]^{1/2}t\}}{4\beta^4+s^4}ds$$

and putting $2\beta^2 t/\gamma^{1/2}=\tau$, $s/(4^{1/4}\beta)=\theta$, we obtain

$$J(t)=j(\tau)=\frac{1}{D\pi(4^{3/4}\beta^3)}\int_0^\infty\frac{\cos[(1+\theta^4)^{1/2}\tau]}{1+\theta^4}d\theta$$

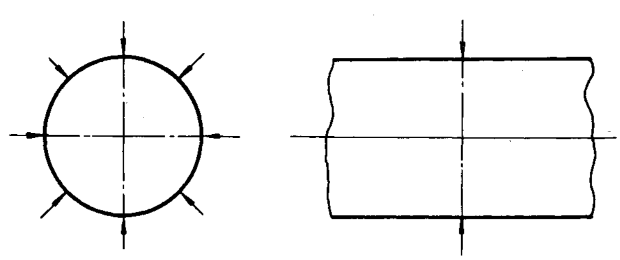

Fig. 1 The shell and the load.

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Differentiating this expression for $j(\tau)$ gives[2]

$$-D\pi(4^{1/4}\beta^3)\frac{dj(\tau)}{d\tau}=\frac{1}{2}\int_1^\infty\frac{\sin\alpha\tau}{(\alpha^2-1)^{3/4}}d\alpha$$

$$=\frac{\sqrt{\pi}}{4}\left(\frac{\tau}{2}\right)^{1/4}\Gamma\left(\frac{1}{4}\right)J_{1/4}(\tau) \qquad (6)$$

with $(1+\theta^4)^{1/2}=\alpha$. Utilizing the integral formula of the Bessel functions[3]

$$\int_0^z t^\mu J_\nu(t)dt=\left[z^\mu\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Big/\Gamma\left(\frac{\nu-\mu+1}{2}\right)\right]$$

$$\sum_{k=0}^\infty\left[(\nu+2k+1)\Gamma\left(\frac{\nu-\mu+1}{2}+k\right)\Big/\Gamma\left(\frac{\nu+\mu+3}{2}+k\right)\right]$$

$$\times J_{\nu+2k+1}(z),\quad Re(\mu+\nu+1)>0$$

and integrating Eq. (6), we gain

$$j(\tau)=j(0)-\frac{\Gamma(1/4)}{2^{15/4}\pi^{1/2}D\beta^3}\int_0^\tau z^{1/4}J_{1/4}(z)dz$$

$$=j(0)-\frac{\Gamma(1/4)\Gamma(3/4)\tau^{1/4}}{2^{15/4}\pi D\beta^3}\sum_{k=0}^\infty\frac{(2k+5/4)\Gamma(1/2+k)}{\Gamma(7/4+k)}\times J_{5/4+2k}(\tau)$$

or

$$J(t)=J(0)-\frac{\Gamma(1/4)\Gamma(3/4)t^{1/4}}{4^{7/4}\pi D\beta^{5/2}\gamma^{1/8}}\sum_{k=0}^\infty\frac{(2k+5/4)\Gamma(1/2+k)}{\Gamma(7/4+k)}$$

$$\times J_{5/4+2k}(2\beta^2t/\gamma^{1/2})$$

Noticing $J(0)=w(0,\infty)$, the final expression for $w(0,t)$ is obtained

$$w(0,t)=\frac{\Gamma(1/4)\Gamma(3/4)t^{1/4}}{4^{7/4}\pi D\beta^{5/2}\gamma^{1/8}}\sum_{k=0}^\infty\frac{(2k+5/4)\Gamma(1/2+k)}{\Gamma(7/4+k)}$$

$$\times J_{5/4+2k}(2\beta^2t/\gamma^{1/2}) \qquad (7)$$

The transient solution for M_x can be developed by following a procedure similar to that for $w(0,t)$. The final outcome is

$$M_x(0,t)=\frac{\Gamma(1/4)\Gamma(3/4)\gamma^{1/8}}{2^{5/2}\pi\beta^{3/2}t^{1/4}}\sum_{k=0}^\infty\frac{(2k+3/4)\Gamma(1/2+k)}{\Gamma(5/4+k)}$$

$$\times J_{3/4+2k}(2\beta^2t/\gamma^{1/2}) \qquad (8)$$

Solutions for $w(x,t)$ and $M_x(x,t)$

In the general case, it seems difficult for one to evaluate the integrals appearing in Eqs. (2) and (3) analytically to put them into explicit forms, and numerical methods must be resorted to. However, the stationary solutions can still be determined as follows

$$w(x,\infty)=\frac{1}{\pi D}\int_0^\infty\frac{\cos sx}{4\beta^4+s^4}ds=\frac{e^{-\beta x}}{8\beta^3D}(\cos\beta x+\sin\beta x) \qquad (9)$$

$$M_x(x,\infty)=\frac{1}{\pi}\int_0^\infty\frac{s^2\cos sx}{4\beta^4+s^4}ds=\frac{e^{-\beta x}}{4\beta}(\cos\beta x+\sin\beta x) \qquad (10)$$

which again agree with those given in Ref. 1 for the static case.

In addition, making use of the stationary phase technique, asymptotic solutions can be developed for small x and large t. The function

$$f(s)=[(4\beta^4+s^4)/\gamma]^{1/2}$$

has a stationary point only at $s=0$, where

$$f'(s)=f''(s)=f'''(s)=0,\quad f^{(4)}(s)=12/2\beta^2\gamma^{1/2}$$

Noting that the stationary point $s=0$ coincides with the lower limit of the definite integrals in Eqs. (2) and (3) and using the stationary phase technique,[4] we have

$$\int_0^\infty\left\{\frac{\cos\{[(4\beta^4+s^4)/\gamma]^{1/2}t\}}{4\beta^4+s^4}\right\}\cos sx\,ds$$

$$\sim\frac{1}{8\beta^4}\int_{-\infty}^\infty\cos\left[\left(\frac{2\beta^2}{\gamma^{1/2}}+\frac{s^4}{4\beta^2\gamma^{1/2}}\right)t\right]ds$$

$$=\frac{1}{8\beta^4}\left\{\cos\left(\frac{4\beta^4t^2}{\gamma}\right)^{1/2}\int_{-\infty}^\infty\cos\left(\frac{ts^4}{4\beta^2\gamma^{1/2}}\right)ds\right.$$

$$\left.-\sin\left(\frac{4\beta^4t^2}{\gamma}\right)^{1/2}\int_{-\infty}^\infty\sin\left(\frac{ts^4}{4\beta^2\gamma^{1/2}}\right)ds\right\}$$

Since[2]

$$\int_{-\infty}^\infty\cos\left(\frac{ts^4}{4\beta^2\gamma^{1/2}}\right)ds=\frac{1}{2}\int_0^\infty\frac{\cos(t\theta/4\beta^2\gamma^{1/2})}{\theta^{3/4}}d\theta$$

$$=\frac{\Gamma(1/4)\cos(\pi/8)(4\beta^4\gamma)^{1/8}}{2^{3/4}t^{1/4}}$$

$$\int_{-\infty}^\infty\sin\left(\frac{ts^4}{4\beta^2\gamma^{1/2}}\right)ds=\frac{1}{2}\int_0^\infty\frac{\sin(t\theta/4\beta^2\gamma^{1/2})}{\theta^{3/4}}d\theta$$

$$=\frac{\Gamma(1/4)\sin(\pi/8)(4\beta^4\gamma)^{1/8}}{2^{3/4}t^{1/4}}$$

the asymptotic solution for $w(x,t)$ takes the form

$$w(x,t)\sim\frac{e^{-\beta x}}{8\beta^3D}(\cos\beta x+\sin\beta x)$$

$$-\frac{\Gamma(1/4)}{2^{13/4}\pi D\beta^3}\left(\frac{\gamma}{4\beta^4t^2}\right)^{1/8}\cos\left(\frac{\pi}{8}-\frac{2\beta^2t}{\gamma^{1/2}}\right) \qquad (11)$$

Similarly, we can work out the asymptotic solution for $M_x(x,t)$ as

$$M_x(x,t)\sim(e^{-\beta x}/4\beta)(\cos\beta x-\sin\beta x) \qquad (12)$$

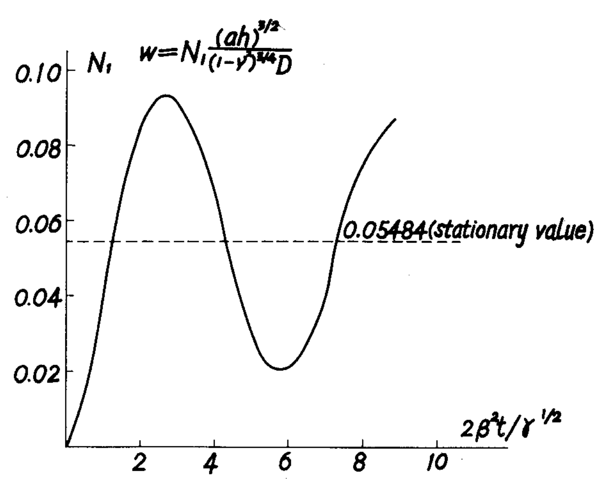

Fig. 2 Variation of $w(0,t)$.

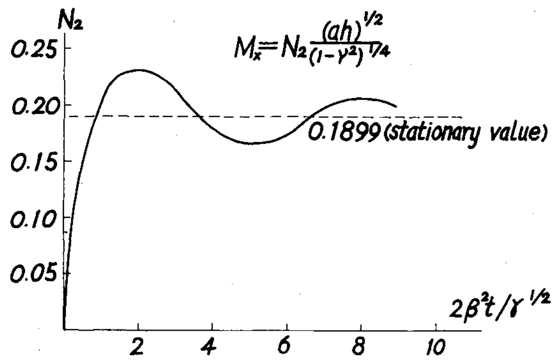

Fig. 3 Variation of $M_x(0,t)$.

which coincides with the corresponding static solution of Eq. (10). Hence, compared to $w(x,t)$, $M_x(x,t)$ approaches its stationary value more rapidly.

Numerical Results and Conclusions

The dynamic responses of w and M_x have been worked out from Eqs. (7) and (8) and presented in Figs. 2 and 3. From the results, the following conclusions can be drawn:

1) For $\infty > t > 0$, w and M_x oscillate with decreasing amplitudes around their stationary values. These regular patterns prove the correctness of the solutions in Eqs. (7) and (8).

2) Compared with w, M_x approaches its stationary value much faster, a conclusion in line with that drawn from the asymptotic solution of Eq. (12).

3) The ratio of maximum to static M_x in the present problem is 1.23 and is of the same order as the dynamic effect coefficients found in some design specifications.[5] However, owing to some idealizing factors involved in the present analysis [such as the idealizing external load $H(t)\,\delta(0)$], the ratio we gain must be somewhat higher than that occurring in the real process.

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Thermoacoustic Convection Heat-Transfer Phenomenon

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Introduction

It is known that rapid heating of a fluid at a boundary (or in a mixing process where chemical reaction takes place) can cause sudden expansion of the fluid near the boundary, giving rise, in turn, to the appearance of pressure waves. These pressure waves, which are of thermal origin, are called thermoacoustics. The appearance of these waves and the ensuing compression and rarefication of the fluid produces a fluid velocity that can result in enhancement of heat-transfer processes by addition of a convective mode. The presence of thermoacoustics has been known for a long time in such configurations as Sondhauss and Rijke tubes, where they can attain frequencies within audible range.[1,2]

One of the main motivations in the investigation of thermoacoustic convection (TAC) heat tansfer is in space application. Manufacturing processes, cryogenic storage, and fluid handling processes in space may all include large TAC heat-transfer rates. Thus in zero-gravity space environment, where it is assumed that conduction is the only heat-transfer mode, TAC heat transfer can play a significant role in the enhancement of heat transfer by introducing a convective mode.

The study of the contribution of the thermally generated pressure waves and the ensuing fluid velocity to heat-transfer processes has been limited primarily to analytical investigations. These studies show that large temperature gradients can produce significant thermoacoustics, giving rise to large heat-transfer rates and short transient times, leading to rapid establishment of steady-state temperature profiles. There has, however, been no experimental investigation of TAC heat transfer in both zero-gravity or gravity environments.

Figure 1 shows the geometrical configuration that was studied in two numerical investigations of a one-dimensional TAC heat transfer.[3,4] Two infinite parallel plates, a distance L apart, that contain an ideal gas are all assumed at an initial temperature T_0. At time $t=0$, the top plate temperature is suddenly increased to $2T_0$. It was found that gas temperature rapidly approaches that of steady state in a very short time. Figure 2 shows the results of the nondimensional temperature profile of helium gas at $x=\frac{1}{2}L$ after 0.2 s. The temperature profile for pure conduction, as well as the linear steady-state results, are also presented in Fig. 2. These results show a very rapid rise of temperature due to the presence of TAC relative to the slow conduction mode. A study of time scales in this one-dimensional TAC heat-transfer problem confirmed that a significant difference between conduction and TAC heating time scales exists.[5]

In order to generate significant thermoacoustic convection heat transfer in a gas, the rate of increase of temperature imposed on the boundary should be large. For the configuration shown, numerical analysis of the governing equations by the authors[6] indicates that thermoacoustic flow velocity is present when the rate of increase of boundary temperature is 22°C/s or larger. For an example of "slow" rise of boundary temperature (approximately a constant rate of 0.7°C/s), which corresponds to Appollo 14 experiments, the numerical solution of the governing equations for a one-dimensional radial model shows no significant contribution of TAC heat transfer[4]; that is, for this slow rate of increase of boundary temperature, the convective heat-transfer mode remains small and conduction effects dominate energy transport to the gas.

In order to verify the importance of TAC heat transfer in zero-gravity and gravity environments relative to pure conduction in fluids, an experimental investigation of a one-dimensional heat-transfer model was initiated under the sponsorship of NASA Lewis Research Center. The preliminary results of this study will be presented following a brief description of the experimental apparatus used in the investigation.

Experimental Study and Results

The experimental apparatus consisted of a cylinder containing air. The cylindrical diameter and height are 0.31 m (1 ft) each. The side walls of the cylinder were insulated to

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